Why might people take on "too much" debt?

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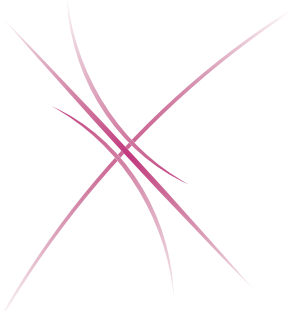

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FOREWORD

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Europe Economics Staff Working Papers are intended to provide a complementary channel for making available economic analysis undertaken by individuals within the firm.

The present paper has been prepared by Dr Andrew Lilico. Andrew is a consultant at Europe Economics. The paper sets out a formal economic framework within which to consider conditions under which people might take on "too much" debt, in some sense.

It is hoped that this work will provide a relevant technical contribution to an important area of current debate.

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1 INTRODUCTION

During 2004 concerns arose in the UK about the total level of household debt, complicating the decisions of the Bank of England Monetary Policy Committee. By June, total debts outstanding of UK individuals were close to £1 trillion. Of this total, consumer credit (e.g. credit card debt) had reached £175bn, and was growing at about 10 per cent per quarter, or 46 per cent per year. Loans secured on dwellings (i.e. mainly mortgages) had reached £818bn, and were growing at about 13 per cent per quarter, or 63 per cent per year.

In 1997 UK household debt was equivalent to about one year's income, but by June 2004 debt was about 35 per cent more than a year's income. The Bank of England Financial Stability Review warned that "The UK household sector debt-to-income ratio has continued to rise rapidly, increasing households' vulnerability to any unexpected rises in interest rates or falls in incomes". A significant concern in the financial press was whether current levels of debt were sustainable and whether rises in interest rates might lead to a significant slowdown in consumption growth.

On the other hand, a Mori poll, also in June 2004, concluded that concerns about credit card debts were over-stated, as only about 5 per cent of credit card users had debts that were significant compared to their incomes — most people's credit card debts were comfortably affordable and 70 per cent paid off their debts rapidly.[1] These results echoed those of an earlier similar survey by Datamonitor.[2]

In our models below we consider first the demand for debt — what determines how much debt households carry, and how much it is sensible ("optimal") for them to carry. We then consider the suppliers of debt — what determines how much debt is optimal for them to provide, and under what circumstances debts may go beyond this level.

As we shall see, our models lead to three key lessons about debt. We shall show that there will be "too much" debt (in senses to be defined below) if:

A: Borrowers over-estimate their future wage increases;

B: Either borrowers or lenders under-estimate future real interest rates — *even if the loans currently being provided would have been paid off by then*;

C: Lenders under-estimate borrower's medium-term wages compared with their later wages.

Section 2 sets out our models. Section 2.1 considers how much debt a borrower will take on when he intends to pay his debts, and conditions under which he might take out an amount of debt he would later regret. Section 2.2 considers the risk that a lender provides so much debt that the borrower defaults, and conditions under which lenders might make mistakes. Section 2.3 considers the case, often (though loosely) discussed in the press where agents are short-sighted in their debt decisions. Section 3 concludes.

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1 "The Credit Card Myth." Mori, June 4th 2004.
2 "UK Plastic Cards 2004", Datamonitory, April 15th 2004.

2 MODEL

Our model will have three periods, $t = 1,2,3$. In each period households discount the future at rate β. Utility is additively separable in periods, and consumers care only about consumption. Households begin period 1 with an endowment E and earn wage w_t in period t. They are able to save s_t in period t (with negative saving interpreted as borrowing). There is an interest rate r_t such that savings s_t are worth $s_t.(1+r_t)$ in period $t+1$. Consumption in period t is given by c_t. Households consume all their remaining wealth in the third period.

Define $-s_1$ as the level of debt the household chooses to bear in period *1*. At the end of period *1*, consumers decide whether to *pay* any debts they carry from period 1, or to *default*. If they *default* then any debts they took on in period *1* are eliminated, but they cannot take on any debts in period *2* (i.e. $s_2 \geq 0$). Denote $\chi \in$ *{pay, default}*.

Thus,

$E + w_1 = c_1 + s_1$ *(1)*

If households do not default then

$s_1.(1+r_1) + w_2 = c_2 + s_2$ *(2)*

Otherwise

$w_2 = c_2 + s_2$ *(2A)*

and

$s_2 \geq 0$ *(2B)*

Either way, for period *3*,

$s_2.(1+r_2) + w_3 = c_3$ *(3)*

Thus, at the start of the problem, the household faces the problem:

$$\max_{\{c_1,c_2,c_3,\chi\}} U^1 = \sum_{t=1}^{3} \beta^{t-1} u(c_t)$$

s.t. *(1), (3), (2)* if χ = *pay*, *(2A)* & *(2B)* if χ = *default*.

There will be a threshold level of savings s_1^* such that above that threshold, it will be optimal for the household to *default*. We shall solve for s_1^* below.

Consider a risk-averse form for the utility function, with

$u'(c) > 0, u''(c) < 0$ (4)

We shall focus mainly on a log form, in which $u(c_t) = ln(c_t)$. This simplifies the presentation considerably and focuses the discussion on the intuitive issues we wish to raise.

Our models concentrate on the consumption-smoothing motivation for saving. Saving to invest or saving for entrepreneurship raised their own special issues which are not treated separately here. One should notice, however, that the point of investment is presumably ultimately consumption, so that investment can perhaps be thought of as subsumed in our models as a risky form of saving.

2.1 When the borrower will pay back his debts

Assume that the *default* condition does not bind (i.e. we assume that in equilibrium the consumer will not want to borrow more than $-s_1^*$). Now work backwards from the problem the household will face in period *2*:

$$\max_{\{c_2,c_3\}} U^2 = \sum_{t=2}^{3} \beta^{t-2} u(c_t)$$

subject to *(2)* and *(3)*, with s_1 given.

Define $U_t = u(c_t)$. Then, since *(3)* makes c_2 a function of c_3 we have

$$\frac{dU^2}{dc_2} = \frac{dU_2}{dc_2} + \beta \frac{dU_3}{dc_3}\frac{dc_3}{dc_2}$$

Hence, with a log utility, from the first-order condition on c_2

$c_2.\beta.(1+r_2) = c_3$ *(5A)*

Similarly

$c_1.\beta.(1+r_1) = c_2$ *(5B)*

(1), (2), (3), and *(5B)* $\Rightarrow$

$$c_1 = \frac{E + w_1 + \frac{w_2}{1+r_1} + \frac{w_3}{(1+r_1)(1+r_2)}}{1+\beta+\beta^2} \qquad (6)$$

Equation *(6)* can be understood intuitively. The top part of the right-hand-side is the Present Value of total lifetime resources, as seen from the beginning of the problem. If the decision-maker were not impatient (i.e. if β were equal to *1*), consumption would be perfectly smoothed (as

happens under log utility) if consumption in each of the three periods were exactly one third of total lifetime resources. But because $\beta < 1$, instead of *3* we divide by the impatient agent equivalent, $(1 + \beta + \beta^2)$.

(6), (1) $\Rightarrow$

$$s_1 = \frac{\left(\beta + \beta^2\right)\left(E + w_1\right) - \frac{w_2}{1 + r_1} - \frac{w_3}{\left(1 + r_1\right)\left(1 + r_2\right)}}{1 + \beta + \beta^2} \qquad (7)$$

From *(7)* clearly (and unsurprisingly) period *1* savings (debt) tends to decrease (increase) as w_2 and w_3 increase for a given level of $E + w_1$, but savings (debt) tends to increase (decrease) as $E + w_1$ rises. In other words, the higher the proportion of income a household anticipates garnering later in its life, the lower the savings (or the higher the debt) it will choose to bear early in its life.

Also clearly, savings are increasing in r_1. Thus, as intuition would suggest, people tend to save more (or take on less debt) at higher interest rates.

Less intuitively, we find a similar result for r_2 — i.e. at higher period 2 interest rates, households will tend to take on less debt in period 1. This means that if holding debt is going to become more expensive later in life, even if we are intending to pay back our debts before then, we should hold less debt today. (The reason is that if real interest rates in period 2 are higher than we expect, our consumption smoothing will not turn out to be as efficient as we have thought— we should have saved more/borrowed less in period 1 so as to have more resources available for consumption smoothing later.)

The r_2 result may be a particular interest, since sometimes high debt levels are defended by suggesting that, even if interest rates rise, today's debt will be "affordable" (since not an overly burdensome proportion of current incomes). In this case, however, that misses the point, since current debts will be repaid by period 2. The effect of interest rate rises here is not that they lead to default. Rather, it is that such rises make high levels of borrowing inefficient from the point of view of managing consumption across the life-cycle. A possible consequence would be that if agents discovered they had over-borrowed, they might quite suddenly reduce their consumption.

We also find that as agents discount the future more (i.e. as β falls) agents will tend to take on more debt, as one would expect.

Thus we have the following two key lessons. Agents will tend to take on more debt than, in retrospect, they would consider optimal,

A: if they over-estimate their future wages (e.g. if w_2 and w_3 do not turn out as high as they expected);

B: if they under-estimate future real interest rates — *even if they would have paid off their current debts by then* (e.g. if r_2 turns out to be higher than they expect).

The first of these is reminiscent the Review's warnings of "unexpected…falls in incomes". The second is reminiscent of the warnings about real interest rates rises, but with the interesting twist that the "affordability" of current debts, when interest rates rise, rather misses the point.

2.2 When borrowers might *default*

Lessons (A) and (B) related to circumstances under which agents will tend to borrow more than *the agents themselves* would consider optimal, in retrospect. Another form of "too much" debt might arise when lenders provide more debt than *the lenders* would consider optimal, in retrospect. One important set of circumstances in which this might occur is when lenders would provide debt that they expected to be paid back, but which wasn't — i.e. in which lenders have under-estimated the probability of default.

In our full information setting, a Rational agent can only be harmed by being able to default (and hence potentially being credit-constrained by lenders), because if the debt level at which he is credit-constrained were really superior to the solution under which he cannot default, he could have chosen that level of debt even in the case where he faced no credit constraint.

However, although our setting involves perfect information, we can easily see why default might be useful once uncertainty is introduced — if an individual significantly over-estimated his future wages, for example, he might take on a level of debt such that, when periods 2 and 3 arrived his consumption (and hence later-life utility) was very low indeed. Under uncertainty, given that he is risk-averse, this might make him very reluctant to take on any significant debt in period 1, reducing bank profits and reducing individual consumption smoothing. In contrast, with default permitted banks bear more of the risk that future wages are not as expected, and their ability to manage such risks might be significantly greater than that of individuals. Potentially, then, under uncertainty the possibility of default, although it might lead to agents being credit-constrained, might benefit both individuals and banks because profits might be higher (the greater volume of loans requested might more-than-offset the occasional bankruptcy) and individual welfare superior (since higher loans might mean more efficient consumption-smoothing).[3]

In our model, borrowers *default* on their period *1* debts if

$$U^2(default) > U^2(pay)$$

for a given level of period 1 debt, $-\overline{s}_1$.

[3] To see this more concretely, consider a case in which, instead of known wages in periods 2 and 3 the potential borrower has wages that might be *W* (with a probability of 99%) or 0 (with a probability of 1%) each time, with the value only becoming known at the start of period 2 (i.e. at the start of period 2 the agent discovers the values of w_2 and w_3). If such an agent carries any debt into period 2 his expected utility is $-\infty$, since if both w_2 and w_3 were to turn out zero (which would be a possibility), his consumption would be 0 in period 3. Hence such an agent will always save in period 1, never borrowing anything. Yet it should be clear that with a distribution of such agents whose wage outcomes were fairly independent, a risk-neutral bank could make profits lending to them (at some appropriate interest rate) if they were allowed to default in the unlikely event that their mid- and late-life wages were both zero, and (for appropriate values of w_1 and *W*) it might then be in the interests of such agents to borrow.

The consequence of default is that it will be impossible to borrow to smooth consumption between periods 2 and 3. This will only be an issue for the agent if he might *want* to borrow to so smooth consumption — i.e. if his period 2 wages are less than his ideal period 2 consumption.

By analogy with the results above, in the case in which agents choose to pay in period 2, we have

$$c_2 = \frac{\bar{s}_1.(1+r_1) + w_2 + \frac{w_3}{1+r_2}}{1+\beta} \qquad (8)$$

For given real wages, real interest rates, and a given discount factor, when $-\bar{s}_1$ is such that c_2 in equation *(8)* is exactly equal to w_2, the agent will be indifferent between defaulting and not defaulting. For any lower level of c_2 (i.e. higher level of $-\bar{s}_1$ relative to the other variables) the agent will be better to default.

Hence, the threshold debt level, $-s_1^*$, at which default will occur is when

$$c_2 = \frac{s_1^*.(1+r_1) + w_2 + \frac{w_3}{1+r_2}}{1+\beta} = w_2$$

$$\Rightarrow -s_1^* = \frac{\frac{w_3}{1+r_2} - \beta.w_2}{1+r_1} \qquad (9)$$

From equation *(9)* we can see that the threshold debt level above which bankruptcy will occur is

- Increasing in w_3 — the higher late-life income is, the more likely the agent will want to borrow in mid-life;
- Decreasing in w_2 — the higher is mid-life income, the less likely the agent will want to borrow at that stage, and hence the lower the penalties for defaulting;
- Decreasing in r_1 — the higher the burden of the debt to be repaid, the more likely is default;
- Decreasing in r_2 — the higher the cost of borrowing in period 2, the less likely the agent is to want to borrow at that stage.
- Decreasing in β — the less the agent discounts the future, the more concerned he will be to be able to borrow to smooth consumption.

Banks have many processes of estimating default probability, often based on characteristics such as profession, age, current debts, and others which might, in terms of the model here, be providing proxies for time preference (β) and lifetime wage profile (w_2 and w_3).

Banks are determining whether to provide debt of $-s_1$ in period 1. From the discussion above we can see that at that stage banks might provide "too much" debt if they were to

- under-estimate w_2;
- over-estimate w_3; or
- under-estimate r_1 or r_2.

We can see that, as above, an under-estimate of how much interest rates will rise in the future *even after current debts are repaid* will tend to lead to "too much" debt. In this case, again, it is not that default is induced because people are unable to repay their current debts. Rather, it is that once they see that interest rates for taking on *future* debt will be higher, they no longer consider it useful to repay their *old* debts. For example, an unanticipated rise in interest rates might lead people to default, even in cases where their current debts were at previously fixed rates (and hence the affordability of current loans would not be directly affected by interest rate rises).

One interesting difference in these results from those above is that, while borrowers will take on "too much" debt in period 1 if they *over*-estimate their wages in period 2, lenders will tend to offer "too much" debt in the reverse case in which they *under*-estimate mid-life wages. The reason is that high mid-life income, relative to late-life income, means that the agent will not be interested in borrowing in mid-life anyway, so that the cost of default is zero.

Hence we have another lesson: Lenders will provide more debt than they would, in retrospect, consider optimal

C: if they under-estimate borrowers' medium-term wages compared with their later wages (e.g. if w_2 turns out higher than lenders expect).

This might suggest that it would be useful to have constraints on an individual's right to default (as well as consequential costs). It is not our purpose here to make precise recommendations about bankruptcy rules, but it might be more efficient if the ability to default depends on some assessment of current wages compared with those in other periods — for example, maybe default should only be permitted in cases where current wages are lower than at the point the loan was taken?[4]

[4] Of course, there are many other ways in which default rights might be restricted. For example, perhaps people should not be able to default on all of their debts, but only on some proportion? Or perhaps default should affect future wage prospects?

2.3 When Borrowers are Short-Sighted

Consider a household considering how much to borrow but being short-sighted (though not naïve).[5] Such a household focuses on its payoffs in the first two period, taking account of the third period, at the start of the problem, only in the sense that it is aware that considerations about the third period will affect its second-period play.

Such a household maximises

$$\hat{U} = \ln(c_1) + \beta.\ln(c_2) \qquad (9)$$

subject to

$$E_1 + w_1 = c_1 + s_1 \qquad (10)$$

and

$$c_2 = \frac{s_1.(1+r_1) + w_2 + \frac{w_3}{1+r_2}}{1+\beta} \qquad (11)$$

Substituting *(10)* and *(11)* into *(9)* we have,

Using, s_1

$$\text{Max } \hat{U} = \ln(E + w_1 - s_1) + \beta.\ln\left(\frac{s_1.(1+r_1) + w_2 + \frac{w_3}{1+r_2}}{1+\beta}\right)$$

$$\text{FOC}(s_1): \quad \frac{1}{(E + w_1 - s_1)} = \frac{\beta.(1+r_1)}{s_1.(1+r_1) + w_2 + \frac{w_3}{1+r_2}}$$

$$\Rightarrow \qquad s_1 = \frac{\beta.(E + w_1) - \frac{w_2}{1+r_1} - \frac{w_3}{(1+r_1)(1+r_2)}}{1+\beta} \qquad (12)$$

Comparing equations (12) and (7), we see that, for positive parameters,

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[5] In this paper we shall concentrate on what our earlier working paper "Regulating Markets with Short-sighted Decision-makers" (2004.1) referred to as "adapted" short-sightedness, also called "limited foresight" in the sense of Jehiel, P. (1995), "Limited horizon forecast in repeated alternate games," *Journal of Economic Theory*, **67**, 497-519.

$$\beta.(E + w_1) < \left(\beta + \beta^2\right)(E + w_1) \qquad (13)$$

Hence, whenever s_1 would be negative in equation *(7)*, so

$$\left(\beta + \beta^2\right)(E + w_1) < \frac{w_2}{1 + r_1} - \frac{w_3}{(1 + r_1)(1 + r_2)}$$

(i.e. the Rational agent would be borrowing in period 1), we have

$$\beta.(E + w_1) < \left(\beta + \beta^2\right)(E + w_1) < \frac{w_2}{1 + r_1} - \frac{w_3}{(1 + r_1)(1 + r_2)}$$

so the short-sighted agent would be borrowing even more. Furthermore,

$$(1 + \beta) < \left(1 + \beta + \beta^2\right)$$

so that the negative expression on the top of the right-hand-side of *(12)* is discounted by less than that in *(7)*.

Hence, in this model whenever a Rational agent would want debt in period 1, a short-sighted agent would want more debt.

Once period 2 is reached, the short-sighted agent will make the Rational assessment of whether or not to default. It is interesting to note that, in contrast to Rational agents (as discussed above), allowing short-sighted agents to default, so that banks credit-constrain them, might leave them closer to their optimal consumption paths than if they were unable to default, because their chosen consumption paths involve taking on more debt in period 1 than is rational. Hence the possibility that agents are short-sighted might be a further rationale for allowing bankruptcy — with short-sighted agents credit constraints can be welfare-improving.

3 CONCLUSION

In this paper we have considered conditions under which agents might take on "too much" debt, in two senses:

i) Borrowers take out more debt than they, in retrospect, would consider ideal;

ii) Lenders provide more debt than is ideal to them, because there will be more bankruptcy than they expect and their loans will never be repaid.

Modelling these senses of "too much debt" leads to three key lessons. There will be "too much" debt if

A: Borrowers over-estimate their future wage increases;

B: Either borrowers or lenders under-estimate future real interest rates — *even if the loans currently being provided would have been paid off by then*;

C: Lenders under-estimate borrower's medium-term wages compared with their later wages.

We have also investigated the effect of short-sightedness on debt decisions, and illustrated that short-sighted agents will tend to take out more debt than they, themselves, would consider optimal in retrospect. This leads to a rationale for allowing default, since credit-constraining short-sighted individuals can take them closer to their optimal Rational consumption paths.